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ABSTRACT

One of the major goals in mathematics education is to teach the ability to solve word problems. This study compared the word problem solving abilities of second-grade students from Taiwan (N=24) with those of American students (N=24). Two tests were used in the problem solving interviews. The comparisons between the performance of the groups on different problem solving subprocesses revealed that children from Taiwan were better than their American counterparts in schematic knowledge only on word problems using addition number sentences with unknown results. The groups did not differ in their semantic or procedural knowledge. Findings suggest that successful word problem solving involves an integration of different kinds of knowledge underlying word problem solving processes. (ASK)

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Mathematical Word Problem Solving Knowledge: Are second-grade students from Taiwan better than students from the United States?

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One of the major goals of mathematics education is to teach students how to solve word problems. Word problems have been included as part of the instrumentation of large-scale cross-national studies (Stevenson, Stigler, Lee, Lucker, Kitamura, & Hsu, 1985; Stigler, Lee, & Stevenson, 1990). These studies have reported that Chinese students in Taiwan significantly outperformed their American counterparts. However, results of these studies failed to reveal how Chinese students and American students performed differently in the kinds of knowledge required for word problem solving. It is difficult to judge if Chinese students in Taiwan were better mathematical problem solvers than their American counterparts without comparing their underlying mathematical word problem solving knowledge.

To complement the findings of previous large-scale international mathematics education research, the present study compared the word problem solving abilities of second-grade students from Taiwan with those of American students. This study addressed the following research question:

> Within an applicable theoretical model, how were second-grade students from Taiwan and the United States similar and different in the kinds of knowledge they had underlying mathematical word problem solving processes?

Mayer developed a framework for analyzing the kinds of knowledge and skills required in mathematical problem solving. This theoretical framework, the Analysis of Mathematical Problem Solving (1992), links many kinds of problem solving knowledge and skills (Figure 1). It has been validated by experimental and cross-national research. His studies suggest multiple approaches to investigate national differences in mathematical word problem solving.

In accordance with Mayer's Analysis of Mathematical Problem Solving (1992), three major kinds of knowledge are involved in different phases of word problem solving - semantic knowledge, schematic knowledge, and procedural knowledge. Semantic knowledge is needed for the first stage of word problem solving - problem translation. If the participants are not competent in the semantic knowledge, they will have difficulties constructing correct mental representations. For the second stage of word problem solving, problem integration, schematic knowledge is required. This schematic knowledge refers to the knowledge that different types of word problems can be solved with the same arithmetic operation. It involves the knowledge of problem type. As proposed by Riley, Greeno, and Heller (1983), there are different types of word problems: Change problems, Combine problems, and Compare problems (Table 1). When the solver knows what type of problem is being solved, the schematic knowledge is activated. This knowledge helps to guide the solvers to distinguish relevant from irrelevant information. If the subjects do not possess appropriate schema for a certain problem, they have

difficulty formulating representations of word problems. For solution execution, procedural knowledge is needed to solve word problems. The procedural knowledge refers to arithmetic operations. Studies of word problems typically analyze the correctness of the problem solutions as end results of children's procedural knowledge. Built upon Mayer's Analysis of Mathematical Problem Solving (1992), the research design of this study included multiple research tasks to investigate the three major kinds of knowledge underlying word problem solving processes possessed by second-grade students from Taiwan and the United States (Table 2). These methodologies included problem retelling for semantic knowledge, problem posing for schematic knowledge, and problem solving for procedural knowledge.

Method

Participants

A total of forty-eight second-grade students participated in the present study, 24 of them from Taiwan, and 24 were from the U.S. These participants were selected from two classrooms in each of the two schools. Two schools from Taiwan and two schools from the U.S. were recruited. This procedure was undertaken in order to minimize the influences of individual teachers' instructional style as well as to permit a better match for students' background characteristics.

Schools in each country were chosen based on socioeconomic characteristics of the students' background in order to match the sampling. The six students from each classroom were chosen on the basis of their scores on a written basic skill test. Students who scored over 75% correct of the question items in each of the three sections were eligible for the interviews. Eighty-seven percent of the students from Taiwan scored above the cutoff score, and 57% from the U.S. were qualified for the individual interview tasks. Parent permission letters were sent out to the parents of students who met the eligibility criterion. The first six students who returned the permission slips with his/her parent's signature were interviewed from each classroom.

Materials

There were two tests in the problem solving interview. The first test, the problem retelling test, included six word problems. These word problems were selected from De Corte et al.'s study of first- and second-grade children's problem representation and solutions (1985) because each type of word problem with unknown quantity in different position was included. The second test included six single-step addition and subtraction number sentences with unknown quantities in different positions. The unknown quantities were represented by a blank square. The sequence of problem presentation was counterbalanced to prevent order effects.

Procedure

The interview consisted of two parts; first, problem retelling and second, problem posing. The same researcher administered the two tests in the same order each time. Before each test started, the researcher told the participant that it was just a mathematics test to understand how they solved and created word problems. The interviews were held in a separate room, a classroom, or an outdoor space provided by the schools, as available. The interviews were videotaped and audiotaped for later coding and analysis.

For semantic knowledge and procedural knowledge, the interviewer read the problems twice to the participant, and the students were asked to solve and then to retell six standard word problems. These six problems included three types: Change, Combine, and Compare. The interviewer would repeat the word problems given that the participant did not understand the problems.

For schematic knowledge, the interviewer read the instruction with two sample problems posed from a number sentence to the child. Afterwards, the interviewer showed the participant one number sentence printed on a card and ask him/her to pose two problems for each number sentence. The participants were given a total of six number sentences with unknown quantities in different positions to make up their own word problems in this task.

The interviewer would repeat the problem given that the participant did not understand the problems. The whole interview, including all three tasks, took each child about 20 to 30 minutes to complete.

Data Analysis

For the individual interviews, the data were transcribed from the videotapes and audiotapes. To assure each student only fell in one and only one cell of the design for two-way chi-square contingency table analysis, the participants were coded as student with 2 correct responses, or 1 and 0 correct responses. The definition of correct responses varied with the nature of the hypothesis of the problem solving task. Two-way chi-square contingency table analysis with an alpha level of .05 and $df = 1$ were conducted manually to compare the performance of the two national groups. Given that obtained X^2 exceeded critical X^2 (.05, 1) = 3.841, the null hypothesis of independence between two national groups would be rejected.

Results

The results are presented in three sections - semantic knowledge, schematic knowledge, and procedural knowledge.

Semantic Knowledge

The present study tested the hypotheses about children's semantic knowledge by comparing the proportions of retelling word problem correctly by the two national groups. This study hypothesized that there would be no significant national differences between the two national groups in the proportion of students who could accurately retell

each kind of word problem. The results supported my hypothesis (Table 3) No significant national variations were found.

Schematic Knowledge

For schematic knowledge, the present study asked students to pose their own word problems from given number sentences to assess young children's schematic knowledge. A problem was considered correct when it accurately modeled the number sentence and it could be classified into one of the defined types of problems such as change, combine, exchange, etc.

It was hypothesized that there would be no national differences between the two national groups in the proportion of students who could pose correct word problems for each number sentence. The results of chi-square contingency table tests (Table 4) on their performance in posing word problems from six given number sentences showed a significant national variation in posing word problems from addition number sentence with unknown quantity in the result position. No significant national differences were found with the other five number sentences.

Procedural Knowledge

The present study hypothesized that there would be no significant national differences in the procedural knowledge between the two national groups in the proportion of students who could correctly solve each type of word problem. The results of chi-square contingency table tests (Table 5) showed that there were no

significant differences in the performance of solving each type of word problem by second-grade students from Taiwan and the United States.

Discussion

The present study assessed three kinds of knowledge underlying three phases of the word problem solving processes according to Mayer's Analysis of Mathematical Problem Solving (1992): semantic knowledge for problem translation, schematic knowledge for problem integration, and procedural knowledge for problem solution execution. I used different individual interview tasks to assess the knowledge needed at each phase. Table 6 summarizes the significant national variations in the kinds of knowledge underlying word problem solving processes possessed by second-grade participants from Taiwan and the United States.

The comparisons between the performance of second-grade students from Taiwan and the United States on different problem solving subprocesses revealed that children from Taiwan were better than their American counterparts in schematic knowledge only on the word problems from addition number sentence with unknown results. The national groups did not significantly differ in their semantic knowledge and procedural knowledge.

For semantic knowledge, the results were consistent with my null hypothesis. They suggest that the two national groups do not differ in their semantic knowledge. The

measure of semantic knowledge in this study showed that students from Taiwan and the United States did not differ in retelling key word problems correctly

For schematic knowledge, which is knowledge of different types of word problems used to distinguish the relevant information from the irrelevant information to formulate a representation. The data from the problem posing task revealed that Chinese students were better than American students in schematic knowledge for problem integration, but only with the easiest type of number sentences, addition number sentence with the unknown quantity in the results position. However, with regards to more difficult number sentences, the two national groups did not perform significantly differently from each other in their schematic knowledge. Although the Taiwanese students had a slight advantage on the easiest number sentence, American students in this study were able to pose word problems which could be categorized into more semantic categories (Taiwan $\underline{M}$ = 11, U.S. $\underline{M}$ = 15. Thus the American students demonstrated equally broad semantic knowledge as their Chinese counterparts. The proportion of posed problems which could be semantically categorized into categories established in prior research was below 50% in both countries. This suggests that problem posing from given number sentences, particularly the number sentences with unknown quantities in the position of subsets and start sets, was a novel or difficult task for most children in both countries. However, the slightly greater accuracy of the Taiwanese students on posing word problems may be due to the fact that they had more practice on posing word problems from the most common

type of number sentence in their textbooks than did their American counterparts. This national variation implies that instruction may enhance children's schematic knowledge.

The third kind of knowledge underlying mathematical problem solving processes is procedural knowledge, that is, the ability to perform arithmetic operations. The frequency of correct answers in the problem retelling task was analyzed. Results for each type of problem showed no significant differences between the performance of the two national groups. This implies that students from Taiwan and the United States have the same level of mastery of skills underlying their problem solving processes. This result is consistent with Stigler et al.'s finding (1990) that reported no significant differences between the performance of children on mathematical achievement tests, in lower grade of elementary school. For second-grade students, there are fewer influences from mathematics instruction on their problem solving performance given that instructional styles may significantly affect students' acquisition of mathematical problem solving abilities.

The findings of my study suggest that successful word problem solving involves an integration of different kinds of knowledge underlying word problem solving processes. The emphasis of mathematics instruction on certain skills may skew children's performance on certain phases of word problem solving. To improve children's mathematical word problem solving abilities, optimal mathematics instruction should stress the mastery of all different kinds of skills, including semantic knowledge, schematic

knowledge, strategic knowledge, and procedural knowledge. The emphasis should not be placed solely on either language-based qualitative understanding skills or mathematically-based quantitative knowledge.

The present study provides new perspectives for our understanding of children's word problem solving processes cross-nationally, which previous large-scale international studies of mathematics achievement were not able to capture. This study is one of the few studies which used multiple research tasks to unveil how young children in Taiwan and the United States are similar and different in their underlying knowledge of mathematical word problem solving processes.

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Figure 1

Mayer's Analysis of Mathematical Problem Solving (1992)

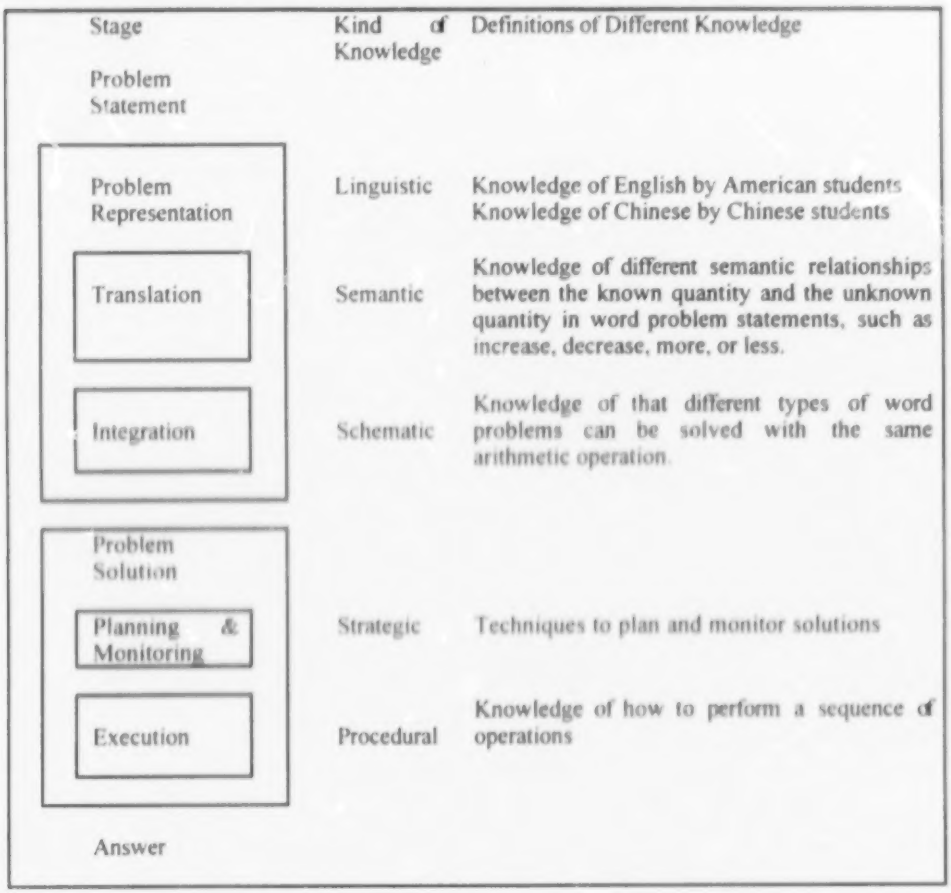
Stage
Kind of Knowledge
Definitions of Different Knowledge
Problem Statement
Problem Representation
Linguistic
Knowledge of English by American students
Knowledge of Chinese by Chinese students
Translation
Semantic
Knowledge of different semantic relationships between the known quantity and the unknown quantity in word problem statements, such as increase, decrease, more, or less.
Integration
Schematic
Knowledge of that different types of word problems can be solved with the same arithmetic operation.
Problem Solution
Planning & Monitoring
Strategic
Techniques to plan and monitor solutions
Execution
Procedural
Knowledge of how to perform a sequence of operations
Answer

Table 1

Semantic Categories of Different Word Problem Types (Riley & Greeno, 1988)

Change/Increase (result unknown) Mary had 3 oranges. Then Tom gave her 5 oranges. How many oranges does Mary have now?	Change/Decrease (result unknown) Mary had 8 oranges. Then she gave 5 oranges to Tom. How many oranges does Mary have now?
Change/Increase (change unknown) Mary had 3 oranges. Then Tom gave her some oranges. Now Mary has 8 oranges. How many oranges did Tom give her?	Change/Decrease (change unknown) Mary had 8 oranges. Then she gave some oranges to Tom. Now Mary has 3 oranges. How many oranges did she give to Tom?
Change/Increase (start unknown) Mary had some oranges. Then Tom gave her 3 oranges. Now Mary has 8 oranges. How many oranges did Mary have in the beginning?	Change/Decrease (start unknown) Mary had some oranges. Then she gave 5 oranges to Tom. Now Mary has 3 oranges. How many oranges did Mary have in the beginning?
Combine/Join (combination unknown) Mary has 3 oranges. Tom has 5 oranges. How many oranges do they have altogether?	Combine/Separate (subset unknown) Mary and Tom have 8 oranges altogether. Mary has 3 oranges. How many oranges does Tom have?
Compare/More (difference unknown) Mary has 3 oranges. Tom has 8 oranges. How many more oranges does Tom have than Mary?	Compare/Fewer (difference unknown) Mary has 8 oranges. Tom has 3 oranges. How many fewer oranges does Tom have than Mary?
Compare/More (compared quantity unknown) Mary has 3 oranges. Tom has 5 more oranges than Mary. How many oranges does Tom have?	Compare/Fewer (compared quantity unknown) Mary has 8 oranges. Tom has 5 fewer oranges than Mary. How many oranges does Tom have?
Combine/More (referent unknown) Mary has 8 oranges. She has 5 more oranges than Tom. How many oranges does Tom have?	Combine/Fewer (referent unknown) Mary has 3 oranges She has 5 fewer oranges than Tom. How many oranges does Tom have?

Table 2

Research Design of the Three Major Kinds of Knowledge Underlying Word Problem Solving Processes Possessed by Second-Grade Students from Taiwan and the United States

Phase	Problem Translation	Problem Integration	Problem Solution Execution
Type of Knowledge	Semantic Knowledge	Schematic Knowledge	Procedural Knowledge
Task Problems	Problem Retelling	Problem Posing	Problem Solving
	Change Join (Start Unknown)	2 + 5 = __	Change Join (Start Unknown)
		6 - 3 = __	
	Combine/Separate (Subset Unknown)	3 + __ = 7	Combine/Separate (Subset Unknown)
		5 - __ = 2	
	Compare/More (difference unknown)	__ + 2 = 8	Compare/More (difference unknown)
		__ - 3 = 6	

Table 3

Results of Chi-Square Contingency Table Tests and Proportions of Students with Correctly Retold Word Problems for Second-Grade Students from Taiwan and the United States in Retelling Word Problems

	Nation		
Retold Statements	Taiwan	U.S.	Statistics
	Change/Join (Start Unknown)		
Students with 2 Correctly Retold Problems	.58	.54	$X^2 = 0.85$, $df = 1$, $p > .05$*
Students with 0 and 1 Correctly Retold Problems	.42	.46	
	Combine/Separate (Subset Unknown)		
Students with 2 Correctly Retold Problems	.75	.79	$X^2 = 0.118$, $df = 1$, $p > .05$
Students with 0 and 1 Correctly Retold Problems	.25	.21	
	Compare/More (Difference Unknown)		
Students with 2 Correctly Retold Problems	.63	.71	$X^2 = 0.375$, $df = 1$, $p > .05$
Students with 0 and 1 Correctly Retold Problems	.37	.29	

Note. There was a total of 24 participants from each nation.

Table 4

Results of Chi-Square Contingency Table Tests and Proportions of Students Who Posed Correct Problems for Second-Grade Students from Taiwan and the United States in Posing Problems from Given Number Sentences

	Nation		Statistics
Posed Word Problems	Taiwan	U.S.	
	2 + 5 = __		
Students with 2 Correct Posed Problems	1.00	.71	$X^2 = 8.195$, $df = 1$, $p < .05$
Students with 0 and 1 Correct Posed Problems	.00	.29	
	6 - 3 = __		
Students With 2 Correct Posed Problems	.83	.62	$X^2 = 2.637$, $df = 1$, $p > .05$
Students with 0 and 1 Correct Posed Problems	.17	.38	
	3 + __ = 7		
Students With 2 Correct Posed Problems	.17	.25	$X^2 = 0.505$, $df = 1$, $p > .05$
Students with 0 and 1 Correct Posed Problems	.83	.75	
	5 - __ = 2		
Students With 2 Correct Posed Problems	.29	.21	$X^2 = 0.444$, $df = 1$, $p > .05$
Students with 0 and 1 Correct Posed Problems	.71	.79	
	__ + 2 = 8		
Students With 2 Correct Posed Problems	.29	.25	$X^2 = 0.105$, $df = 1$, $p > .05$
Students with 0 and 1 Correct Posed Problems	.71	.75	
	__ - 3 = 6		
Students With 2 Correct Posed Problems	.33	.29	$X^2 = 0.097$, $df = 1$, $p > .05$
Students with 0 and 1 Correct Posed Problems	.67	.71	

Note. There was a total of 24 participants from each nation.

Table 5

Results of Chi-Square Contingency Table Tests and Proportions of Students with Correct Written Answers for Second-Grade Students from Taiwan and the United States in Solving Word Problems

	Nation		
Answers	Taiwan	U.S.	Statistics
	Change/Join (start unknown)		
Students with 2 Correct Answers	.38	.33	$X^2 = 0.91$, DF= 1, P > 0.05
Students with 0 and 1 Correct Answer	.62	.67	
	Combine/Separate (subset unknown)		
Students with 2 Correct Answers	.88	.75	$X^2 = 1.231$, DF= 1, P > 0.05
Students with 0 and 1 Correct Answer	.12	.25	
	Compare/More (difference unknown)		
Students with 2 Correct Answers	.96	.79	$X^2 = 3.048$, DF= 1, P > 0.05
Students with 0 and 1 Correct Answer	.04	.21	

Note. There was a total of 24 participants from each nation.

Table 6

Summary Table of Significant National Variations in Three Kinds of Knowledge Underlying Word Problem Solving Processes Possessed by Second-Grade Students from Taiwan and the United States.

Phase	Problem Translation	Problem Integration	Problem Solution Execution
Type of Knowledge	Semantic Knowledge	Schematic Knowledge	Procedural Knowledge
Task	Problem Retelling	Problem Posing	Problem Solving
Results	The two national groups had equal accuracy in retelling problems.	Chinese students performed better on the easiest number sentence. The two national groups were equal in breath of schematic knowledge.	Chinese and American students had equal accuracy in solving the three types of word problems.